Acta Morphologica Generalis
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AMG Vol.1 No.2 (2012)
ISSN 2001-2241

On the Formal Properties of Morphological Models[*]

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Abstract: This article examines the formal (mathematical/combinatorial) properties of morphological models and examines the relationship between these formal properties and the empirical contents of morphological models. Three central combinatorial relationships (in the form of ratios) are explored in order to ascertain if such models can by "typed" according to these formal properties. A number of models are compared on the basis of these ratios and their divergences plotted.

Keywords: general morphological analysis; cross-consistency assessment; formal properties of models; modelling theory.

1. Introduction

When we create models, whether these are quantified or non-quantified, we construct them on a scaffolding of *dimensions* – i.e. mental constructs which support a range of values or conditions. Together, these dimensions define a conceptual space. Spaces have certain properties, including relationships of *connectivity* between dimensions (topology), and relationships of *measure* within and between the values ranges of those dimensions (geometry). (For more details see Ritchey, 2012.)

The relationships within a conceptual space are dependent upon the nature of the concepts involved in *defining* the space. In the case of morphological spaces (or models) this is self-evident. Here dimensionality is not expressed in the form of continuous variables (as we do with physical space) but in the form of variables with well-defined, finite, discrete value ranges. And in such a discrete manifold, the principles of its *internal relations* are already implied by the specification of the dimensions of the manifold, and through the logical, empirical and normative constraints placed on them by *real world problems*. This makes up the all important *content* of the model. In comparison, the *mathematical*/*combinatorial* properties of morphological models are only a formal aspect of this *content*.

However, we cannot help but ask the question of whether there is any meaningful relationship between the formal properties of a morphological model and its empirical contents. This is a valid question which, we feel, warrants the study of these formal properties – on at least two grounds:

Firstly, if it were possible to classify morphological models into different "types" on the basis of purely formal characteristics, this might help us to better understand morphological modelling in general. There is also a matter of pure academic curiosity: It would be intriguing to see to what extent morphological models/spaces *can*, in fact, be ascribed metric and topological properties *analogous* to the general concept of "space" originally discussed by Bernhard Riemann (1953) and, more recently, to the conceptual spaces ("geometry of cognitive representations") studied by Gärdenfors (2004).

This article is divided up into the following sections:

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[*] This is an extended, stand-alone version of Chapter 7 in Ritchey (2011).

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In Section 2, a short background to General Morphological Analysis (GMA) is presented – for those readers who are new to this area. NOTE: *For those who already have a good working knowledge of general morphological modelling, you can skip this section and go on to Section 3.*

In Section 3, we will look at some of the purely *combinatorial aspects* of morphological models, such as how the number of dimensions and the number of parameter values determines the size of the problem space and the cross-consistency matrix.

Section 4 will take a look at a number of *relationships* which are dependent upon the empirical contents of the model, namely: how the *global connections between parameters* ("topology") and the internal relationships between parameters pairs ("geometry") influence the size and form of the solution space of the model.

Finally, a number of client-based morphological models have been selected and compared on the basis of these derived relationships.

2. Background to General Morphology*

The term *morphology* derives from antique Greek (*morphê*) which means *shape* or *form*. Morphology is "the study of form or pattern", i.e. the shape and arrangement of parts of an object, and how these *conform* to create a *whole* or Gestalt. The "objects" in question can be physical (e.g. an organism or an ecology), social/organizational (e.g. a corporation or a defense structure), or mental (e.g. linguistic forms or any system of ideas).

The first to use the term *morphology* as an explicitly defined scientific method would seem to be J.W. von Goethe (1749-1832), especially in his "comparative morphology" in botany. Today, morphology is associated with a number of scientific disciplines where *formal structure* is a central issue, for instance, in linguistics, geology and zoology.

In the late 1940's, Fritz Zwicky, professor of astrophysics at the California Institute of Technology (Caltech) proposed a *generalized form of morphology*, which today goes under the name of General Morphological Analysis (GMA)

> "Attention has been called to the fact that the term *morphology* has long been used in many fields of science to designate research on structural interrelations – for instance in anatomy, geology, botany and biology. ... I have proposed to generalize and systematize the concept of morphological research and include not only the study of the shapes of geometrical, geological, biological, and generally material structures, but also to study the more abstract structural interrelations among phenomena, concepts, and ideas, whatever their character might be." (Zwicky, 1969, p. 34)

Zwicky developed GMA as a method for structuring and investigating the total set of relationships contained in multi-dimensional, non-quantifiable, problem complexes (Zwicky 1966, 1969). He applied the method to such diverse fields as the classification of astrophysical objects, the development of jet and rocket propulsion systems, and the legal aspects of space travel and colonization. He founded the Society for Morphological Research and championed the "morphological approach" from the 1940's until his death in 1974.

Morphological analysis was subsequently applied by a number of researchers in the USA and Europe in the fields of policy analysis and futures studies (e.g. Taylor,1967; Ayres, 1969; Rhyne, 1971; Müller-Merbach, 1976; Godet, 1994; Coyle & McGlone, 1995; Ritchey, 1997). In 1995, advanced computer support for GMA was developed at the Swedish Defence Research Agency FOI in Stockholm.

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* For a more detailed presentation, see the JORS article:" Problem Structuring with Computer-Aided Morphological Analysis" (pdf) at: http://www.swemorph.com/pdf/psm-gma.pdf.

This has made it possible to create non-quantified inference models, which significantly extends GMA's functionality and areas of application (Ritchey, 1998-2012). Since then, some 100 projects have been carried out using GMA, for structuring complex policy and planning issues, developing scenario and strategy laboratories, and analyzing organizational and stakeholder structures.*

Essentially, GMA is a method for identifying and investigating the total set of possible relationships or "configurations" contained in a given problem complex. This is accomplished by going through a number of iterative phases which represent cycles of analysis and synthesis – the basic method for developing (scientific) models (Ritchey, 1991).

The method begins by identifying and defining the most important dimensions (or *parameters*) of the problem complex to be investigated, and assigning each dimension a range of relevant *values* or *conditions*. This is done mainly in natural language, although abstract labels and scales can be utilized to specify the set of elements defining the discrete *value range* of a parameter.

A morphological field is constructed by setting the parameters against each other in order to create an n-dimensional configuration space (Figure 1). A particular *configuration* (the darkened cells in the matrix) within this space contains one "value" from *each* of the parameters, and thus marks out a particular state of, or possible formal solution to, the problem complex.

Parameter A	Parameter B	Parameter C	Parameter D	Parameter E	Parameter F
Condition A1	Condition B1	Condition C1	Condition D1	Condition E1	Condition F1
Condition A2	Condition B2	Condition C2	Condition D2	Condition E2	Condition F2
Condition A3	Condition B3	Condition C3		Condition E3	Condition F3
Condition A4	Condition B4	Condition C4		Condition E4	Condition F4
Condition A5		Condition C5		Condition E5	
				Condition E6	

Figure 1: A 6-parameter morphological field. The darkened cells define one of 4800 possible (formal) configurations.

The point is, to examine all of the configurations in the field, in order to establish which of them are possible, viable, practical, interesting, etc., and which are not. In doing this, we mark out in the field a relevant *solution space*. The solution space of a Zwickian morphological field consists of the subset of all the configurations which satisfy some criteria. The primary criterion is that of internal consistency.

Obviously, in fields containing more than a handful of variables, it would be time-consuming – if not practically impossible – to examine all of the configurations involved. For instance, a 6-parameter field with 6 conditions under each parameter contains more than 46,000 possible configurations. Even this is a relatively small field compared to the ones we have been applying.

* For a list of projects, see http://www.swemorph.com, u/Project List

Thus the next step in the analysis-synthesis process is to examine the *internal relationships* between the field parameters and "reduce" the field by weeding out configurations which contain mutually contradictory conditions. In this way, we create a preliminary outcome or solution space within the morphological field without having first to consider all of the configurations as such.

This is achieved by a process of *cross-consistency assessment*. All of the parameter values in the morphological field are compared with one another, pair-wise, in the manner of a cross-impact matrix (Figure 2). As each pair of conditions is examined, a judgment is made as to whether – or to what extent – the pair can coexist, i.e. represent a consistent relationship. Note that there is no reference here to direction or causality, but only to mutual consistency. Using this technique, a typical morphological field can be reduced by up to 90 or even 99%, depending on the problem structure.

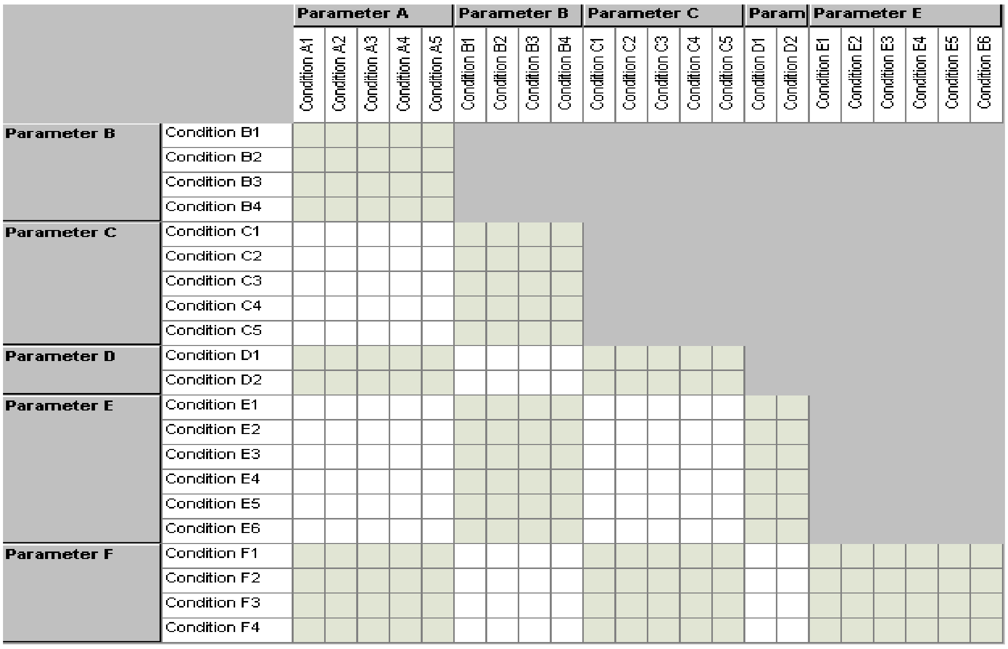


Figure 2: The cross-consistency matrix for morphological field in Figure 1.

There are three principal types of inconsistencies involved here: purely *logical* contradictions (i.e. those based on the nature of the concepts involved); *empirical* constraints (i.e. relationships judged be highly improbable or implausible on practical, empirical grounds), and *normative* constraints (although these must be used with great care, and clearly designated as such).

This technique of using pair-wise consistency assessments between conditions, in order to weed out internally inconsistent configurations, is made possible by a principle of dimensionally inherent in morphological fields, or any discrete configuration space. While the number of configurations in such a space grows exponentially with each new parameter, the number of *pair-wise relationships between parameter conditions* grows only in proportion to the triangular number series – a quadratic polynomial. Naturally, there are also practical limits reached with quadratic growth. The point, however, is that a morphological field involving as many as 100,000 formal configurations can require no more than few hundred pair-wise evaluations in order to create a solution space.

When this solution (or outcome) space is synthesized, the resultant morphological field becomes an *inference model*, in which any parameter (or multiple parameters) can be selected as "input", and any others as "output". Thus, with dedicated computer support, the field can be turned into a laboratory with which one can designate initial conditions and examine alternative solutions.
GMA seeks to be integrative and to help discover new relationships or configurations. Importantly, it encourages the identification and investigation of boundary conditions, i.e. the limits and extremes of different parameters within the problem space. The method also has definite advantages for scientific

communication and – notably – for group work. As a process, the method demands that parameters, conditions and the issues underlying these be clearly defined. Poorly defined concepts become immediately evident when they are cross-referenced and assessed for internal consistency. Like most methods dealing with complex social and organizational systems, GMA requires strong, experienced facilitation, an engaged group of subject specialists and a good deal of patience.

3. The formal combinatorial properties of morphological models

3.1 The morphological field

P1	**P2**	**P3**	**P4**	**P5**
P_1V_1	P_2V_1	P_3V_1	P_4V_1	P_5V_1
P_1V_2	P_2V_2	P_3V_2	P_4V_2	P_5V_2
P_1V_3	P_2V_3	P_3V_3	P_4V_3	P_5V_3
P_1V_4		P_3V_4		P_5V_4

Figure 3: Reference morphological field

Let N = number of parameters in a morphological field (in the Reference field, figure 3, N=5) and let P denote a Parameter such that the parameters in a morphological field are:

$$P_1, P_2, P_3 \ldots P_N$$

Let v_x = the number of conditions in the value range of a given parameter P_x, such that the total morphological field is (quantitatively) defined by:

$$\left\{P_x v_i\right\}_{x,i}$$

Then, the total number of *simple configurations* T_{SC} (i.e. a configuration with *one and only one condition designated under each parameter*) in a morphological field is:

$$T_{SC} = \quad v_1 * v_2 * v_3 \ldots v_N$$

or

$$T_{SC} = \prod_{i=1}^{n} v_i$$

This simply shows that T_{SC} increases in a *factorial* manner with the increase in the number of parameters "N". So much for the basic morphological field.

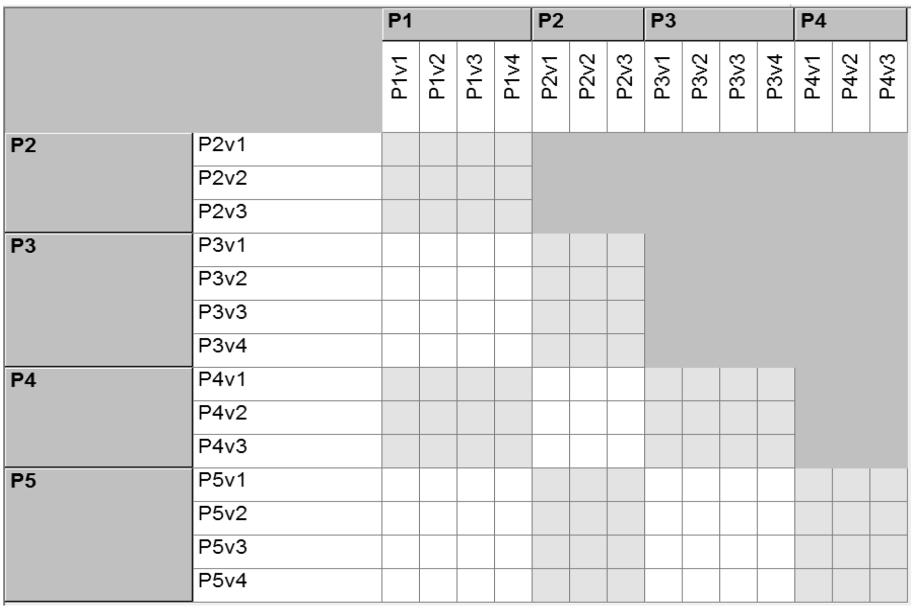


Figure 4: Cross-consistency matrix (CCM) for morphological field in Figure 3.

3.2 The Cross-consistency matrix and Parameter blocks

The Cross-consistency matrix (CCM) pairs off every condition in each parameter with every other condition in all the other parameters. A *parameter block* (PB) consists of all of the paired conditions between two parameters, cross-referenced in the form of a 2-dimensional typology. In Figure 4, the parameter blocks are shown in alternating shaded and white groups.

If N = number of parameters in a morphological field, then the number Parameter Blocks in the field's Cross-Consistency Matrix is:

$$\tfrac{1}{2}N\,(N-1)$$

This is an interesting mathematical expression that pops up all over that place. For instance:

- It is the formula for generating the triangular number series
- It is the possible number of (non-directed) edges connecting N nodes in a graph.
- It is taught in social network theory and in facilitator training as the number of communication channels or possible (two-person) dialogues between N participants in a workshop (which is why group dynamics changes dramatically at around 7-8 people).

Of course, all this has a common base: Generally, ½N(N-1) is the number of dyadic (pair-wise) relationships between N elements or objects. It is equal to the binomial coefficient:

$${}_{n}C_{k} \equiv \frac{n!}{(n-k)!\,k!}$$

when $k = 2$.

Also, ½N(N-1) is central to the discussion of any metric space: it is *the number of coefficients (or functions of position) required to define the metric properties of a space of N dimensions.*(Riemann, 1953).

Number of cross-consistency pairs

If the number of parameters in a morphological model is N and the number of parameter values for a parameter P_x is v_x, then the number of dyadic (pair-wise) relationships (Ct) between *all parameter values* (and thus the total number of cells in the cross-consistency matrix – CCM) is:

$$\text{Ct} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} v_i \cdot v_j$$

The take-home message is this: that while the number of formal configurations in a morphological model increases "geometrically" (factorially) with each additional parameter, the number of cross-consistency pairs increases "only" in proportion to the quadratic polynomial f(x)=½x(x-1). This is what makes it possible to employ Cross-Consistency Assessment (CCA) to reduce a relatively large problem space to a more manageable solution space, without having to examine every configuration in the problem space.

To sum up: we have four magnitudes which determine the primary formal properties of a morphological model:

N	= number of parameters
½N(N-1)	= number of parameter blocks in the CCM
ΣΣ vv	= number of pair-related cells in the CCM
Π v	= total number of simple configurations in the model

In the case of v = 4 for each of the parameters, the relationship between these magnitudes is:

a	*b*	*c*	*d*
N	½N(N-1)	$\sum_{i=1}^{n-1} \sum_{j=2}^{n} v_i \cdot v_j$	$\prod_{i=1}^{n} v_i$
Number of parameters	Number of dyadic relationships between parameters blocks	Number of CCM cells	Number of simple configurations
2	1	16	16
3	3	48	64
4	6	96	256
5	10	160	1024
6	15	240	4096
7	21	336	16348
8	28	448	65536
9	36	576	262144

Table 1. The primary formal properties of a morphological model (for v=4)

4. The relationships between form and content

4.1 Three ratios

Expressions *b*, *c* and *d* (Table 1) are formally determined by N and V_x, i.e. the number of parameters and the number of conditions under each of the parameters. There are three other quantities that are determined by the *logical, empirical and normative judgements* made in the Cross-Consistency Assessment (CCA) which, together with *b*, *c* and *d*, give rise to three ratios that can help us to formally "type" morphological models.

These three ratios are:

1. The ***connectivity quotient (κ -** Kappa**)***: The ratio of the number of parameters blocks *which are constrained* (PBc) to the *total* number of parameter blocks ½N(N-1). This is analogous to how the dimensions of an abstract space are topologically connected.

2. The ***consistency quotient*** (**χ -** *χĩ /Chi*): The ratio of the number of *mutually constrained parameter value pairs* in the Cross Consistency Matrix (CCM) to the *total number of parameter value pairs* (or *cells*) in the CCM.

3. The ***solution space quotient*** (**ϛ -** *Stigma*): The ratio of the number of simple configurations in the *solution space* to the number of simple configurations in the in the *total problem space*.

4.1.1. Connectivity Quotient (κ - Kappa)

Connectedness in a morphological model concerns how the different *dimensional constructs* of the model (i.e. its parameters) "hang together", i.e. are topologically connected. There are two (principal) possibilities here for each of the ½N(N-1) parameter pairs: either two given parameters contain mutual (logical and/or empirical) *constraints*, or they are (logically and/or empirically) *orthogonal*.

Orthogonal means "at right angles". This means that the value ranges of two orthogonal parameters are independent of each other, i.e. they do not interfere with or constrain one another. Since we relate values by way of mutual consistency, then in a pair of orthogonal parameters P_a and P_b, any value of P_a is consistent with (can co-exist with) any value of P_b. Figure 5 shows an orthogonal parameter pair (block), in which the assessment key "-" means: "is consistent with..." or "can co-exist with ...".

		Pa			
		a1	a2	a3	a4
Pb	b1	-	-	-	-
	b2	-	-	-	-
	b3	-	-	-	-
	b4	-	-	-	-

Figure 5: Orthogonal parameter block

An orthogonal relationship between two parameters does not necessarily mean that there is no meaningful *content* associated with the value relations. It simply means that there are no mutual constraints between the parameters; i.e. *everything goes*. However, if a parameter P_k is orthogonal to *all of the other parameters in a morphological model*, then its variability has no effect on the rest of the model. Such a parameter is – so to speak – exogenous to the model as such.

Parameter pairs are *mutually constrained* – and thus "connected" – when *at least* one value pair in the parameter block is deemed inconsistent, impossible or unviable. For instance, if we pit a range of age intervals in a population against a range of body weight intervals, then obviously (for us *homo sapiens*), there are going to be some expected constraints between age values and weight values (as seen in Figure 6). In this hypothetical example, for instance, "X" could mean *highly unlikely* and "?" *pretty extreme*. The diagonal area from bottom right to top left (containing "-") we could call the *main sequence* of the relationship. This type of pattern often turns up when scales are pitted against each other.

		Weight (Kg)			
		< 20	20-50	50-100	> 100
Age (Yrs)	< 5	-	-	?	X
	5-10	-	-	-	?
	10-20	?	-	-	-
	> 20	X	?	-	-

Figure 6: Constrained parameter block.

If the number of parameters in a morphological model is N, then the *minimum number* of connections for the model to hang together as a whole (since every parameter must be connected to at least one other parameter) is N-1. Thus an N-dimensional model is called *minimally connected* when it has exactly N-1 connections and each parameter is *connected to at least one other* parameter. (In graph theory, these minimally connected configurations are called *labelled free trees*.) A model is *completely connected* when it has ½N(N-1) connections, and where every parameter is connected to every other parameter. While there is only one way to *completely connect* a model, there are many ways to *minimally connect* a model. For a model of N parameters there are N^{N-2} distinct minimally connected configurations (i.e. free trees).

Number of parameters N	Minimal connections N-1	Total connections N(N-1)/2	Number of ways to produce minimally connected models N^(N-2)
2	1	1	1
3	2	3	3
4	3	6	16
5	4	10	125
6	5	15	1296
7	6	21	16807
8	7	28	262144
9	8	36	4782969

Table 2: Relationships of connectivity for an N-dimensional morphological model. There are N^{N-2} distinct labelled free trees on N vertices (Cayley's theorem). This table has been corrected from an early version showing
See: https://en.wikipedia.org/wiki/Graph_enumeration

The number of labeled n-vertex undirected graphs is $2^{n(n-1)/2}$
The number of labeled n-vertex free trees is n^{n-2} (Cayley's formula).

The connectedness between two parameters is not *directional*, but simply denotes that two parameters constrain or interfere with each other somehow. Thus the connections between parameters in a morphological model can be represented in the form of an *undirected graph* (which is why the term “connectedness” was originally used in this context). For instance, a maximally connected 4-dimensional model is represented in the following undirected graph (Figure 7):

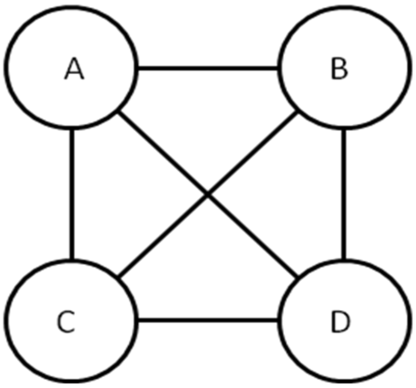


Figure 7: Completely connected model: N = 4; ½N(N-1) = 6.

Similarly, too possible minimally connected 4-dimensiinal models are represented in Figures 8 and 10, with their corresponding CCA matrices shown in figure 9 and 11 (the connected parameter blocks containing “X”, the unconnected blocks containing a *hyphen*).

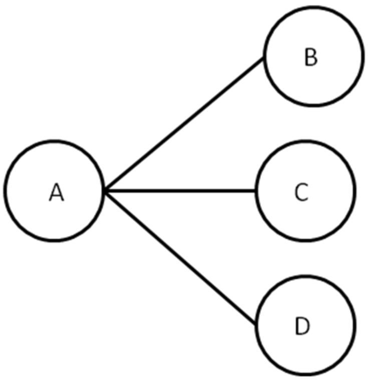


Figure 8: Minimally connected model: N = 4; N-1 = 3.

	A	B	C
B	X		
C	X	-	
D	X	-	-

Figure 9: CCA format for Figure 8. “X”=connected; “-“ = unconnected.

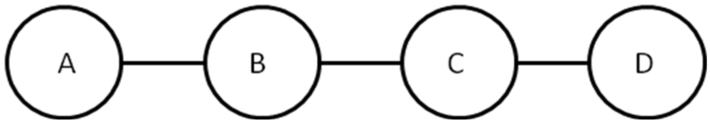


Figure 10: Minimally connected model: N = 4; N-1 = 3.

	A	B	C
B	X		
C	-	X	
D	-	-	X

Figure 11: CCA format for Figure 10. "X"=connected; "-" = unconnected.

The *Connectivity Quotient* **κ** is the ratio of the number of *constrained parameter blocks* (PBc) to the *total number of parameter blocks* ½N(N-1).

$$\kappa = \frac{\text{PBc}}{\tfrac{1}{2}N\,(N-1)}$$

Since the minimum number of Constrained Parameter Blocks (PBc) required in order to define a proper model is (N-1), then the possible range of PBc is:

$$(N-1) \rightarrow \tfrac{1}{2}N(N-1)$$

and where **κ** ranges from: $2/N \rightarrow 1$

4.1.2 The Consistency Quotient (χ) (Chi)

The c*onsistency quotient* is the ratio of the number of mutually constrained (i.e. inconsistent) cells (Cx) in the Cross-Consistency Matrix (CCM) to the total number of cells (Ct) in the CCM.

$$\chi = \text{Cx}/\text{Ct}$$

where $$\text{Ct} = \sum_{i=1}^{n-1}\sum_{j=2}^{n} v_i \cdot v_j$$

The number of pair-wise mutually constrained cells (Cx) in a cross-consistency matrix is determined by the judgements made by the *subject specialist group* doing the morphological modelling. It is an "empirical" input, in the sense that it is not determined by any formal properties of the model. Rather, it is determined by the explicit or implicit *nature of the concepts* supplied in order to create the model. In order to determine Cx**,** one simply has to count them in the CCM.

4.1.3 The Solution space quotient (ς = Stigma)

The *solution space quotient* ς is the ratio of the number of simple configurations making up the *solution space* ($\text{Config}_{\text{sol}}$) to the total number of (formal) simple configurations in the in the *problem space*.

$$\varsigma = \text{Config}_{\text{sol}} \ \textbf{[divided by]} \ \prod_{i=1}^{n} V_i$$

This ratio is a crucial indicator of the nature of the model thus developed. If, for instance, the ratio expressed by ς is near or equal to 1, then the model is hyper-coherent. This means that just about everything is consistent with everything else. There is nothing wrong with such models; they are simply telling us that practically everything is possible. Certain types of futures scenarios models are of this form. If, on the other hand, ς is very small, then the model is hyper-constrained, meaning that very few model configurations are possible. This is also an interesting outcome.

4.2 The relationships between the three ratios

Intuitively we would expect there to be a pattern between these relationships, since, clearly, κ and χ should influence the size of the solution space, and thus the magnitude of ς. Ultimately, we want to see how these relationships express themselves in different types of morphological models with different empirical contents.

The only way to do this is to select and collate a number of actual morphological models developed in real settings for real problems. We have selected 16 representative models, chosen for their breadth of application. For the moment, the *purpose* of these models, i.e. their content, is not of any relevance. We simply want to see the spread of their formal characteristics as concerns the three cited ratios.

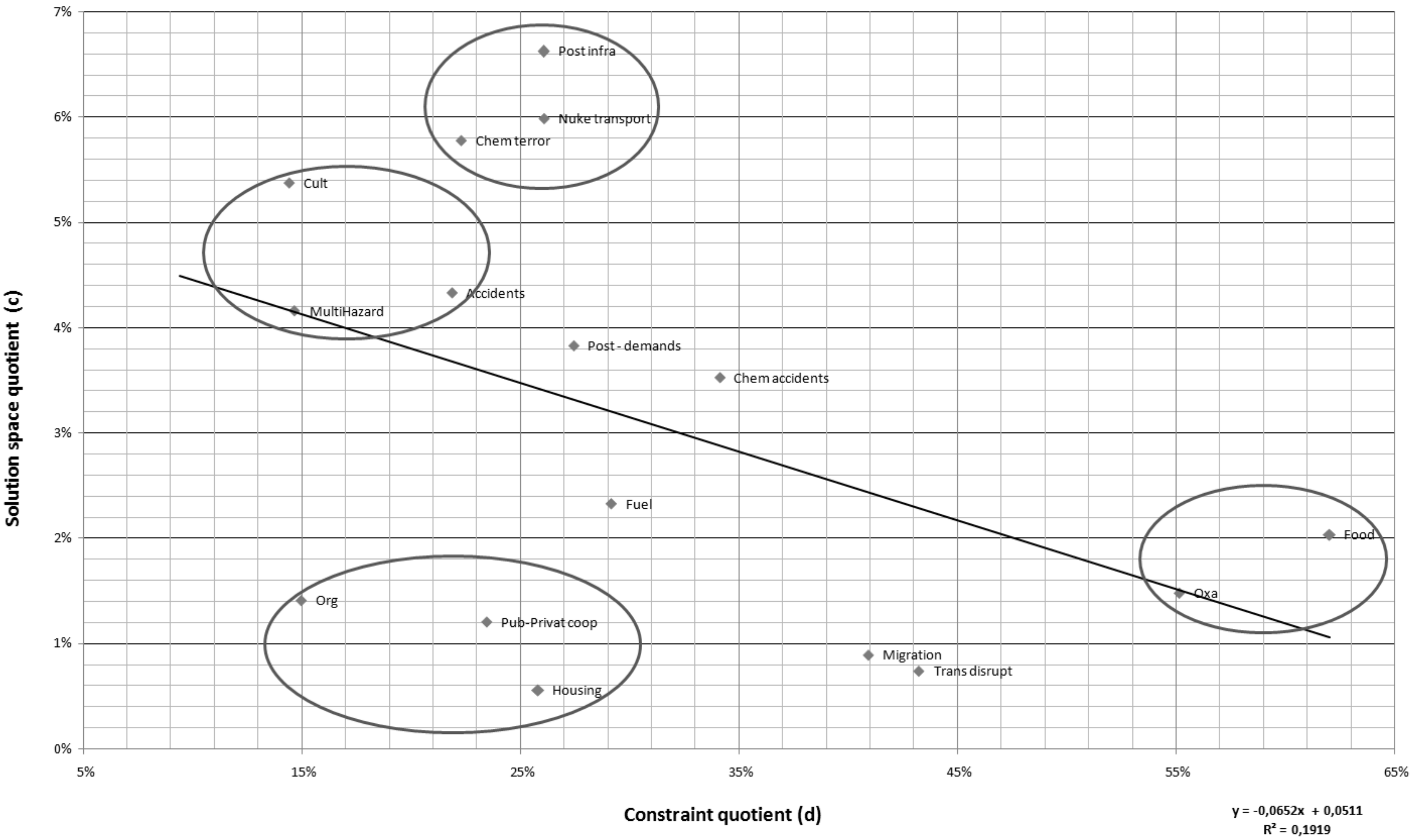
The collation is shown in Graph 1.

The graph is based on the idea that the relationship that best represents the formal constraining properties represented by the cross-consistency assessments (CCA) is: χ **[divided by]** κ, which we shall call the *constraining quotient.* This ratio expresses – in an approximate way – the distribution of the constraints over the whole cross-consistency matrix. The hypothesis is, that for a given χ, the constraints on the model will tend to be greater if these are concentrated to only a few parameter blocks, than if they are spread out more evenly over the entire matrix. Therefore, there should be some correlation between the constraining quotient (χ **[divided by]** κ), and the solution quotient ς which represents the relative size of the solution space.

The graph shows a sequence of models where ς is plotted against (χ **[divided by]** κ, the constraining quotient). It shows a clear inverse relationship between the two, *but not a very strong one*. That there is such an inverse relationship is clear enough: models cannot deviate too far towards the upper right hand side of the graph, as this would represent a highly constrained CCM producing a relatively large solution quotient – an obvious contradiction. It is the same story in the opposite direction: weakly constrained CCM:s would not be expected to produce very small ς.

However, beside this purely combinatorial principle, the divergence of the models from the linear trend fit tells me that the formal aspects of morphological spaces and the *empirical contents of the models* have little to do with each other – at least as concerns the type of studies I have been carrying out. What is needed at this point is a careful comparison of the "divergent" models (e.g. those circled in Graph 1) in order to see what it is that determines this divergence. Experience tells me that two factors should contribute to such divergences: the scaling properties (including non-ordinal value ranges) employed in the dimensions of different models; and differences in the proportions of OR-list and AND-lists employed in different models. But this will have to wait for another time, if not another life.

I hope that Fritz would have enjoyed this formal analysis of his brainchild.



Graph 1: Correlation between *constraint quotient* and *solution space quotient* for 16 morphological models.

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Acknowledgements: Thanks to Johan Schubert, mathematician/operational analyst at FOI, for scrutinizing this text, cleaning up the mathematical notation and providing valuable suggestions. All remaining faults are my own.